An Open System with Spontaneous PT-Symmetry Breaking

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Background

The Hamiltonian of a system characterizes how quantum states evolve with time: $i\frac{\partial\psi}{\partial t} = H\psi$

- Eigenstates are states φ such that $H\varphi \propto \varphi$.
- Imaginary parts of the Hamiltonian's diagonal elements can be viewed as gain/loss in an open optical system.
- *Exceptional points* (EP) are times when some of the eigenstates of the Hamiltonian are parallel.

Parity-time (PT) reversal changes the sign of space and time in a physical process in the following way [1]:

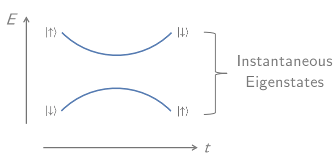
$$x \xrightarrow{\mathcal{PT}} -x \qquad p \xrightarrow{\mathcal{PT}} p \qquad i \xrightarrow{\mathcal{PT}} -i$$

- A state or Hamiltonian has a *PT symmetry* if it is invariant under PT reversal.
- *Spontaneous PT-symmetry breaking* occurs in a system when the Hamiltonian has PT symmetry but its eigenstates do not.

Landau-Zener (LZ) Effect: a system with the Hamiltonian

$$H(t) = \begin{pmatrix} \delta t & \omega_0 \\ \omega_0 & -\delta t \end{pmatrix} \qquad \delta, \omega_0 \in \mathbb{R}$$

- A model for dynamical transition between two states.
- For each time t we may find two instantaneous eigenstates of $H(t)$, with energies $E(t) = \pm\sqrt{\omega_0^2 + \delta^2 t^2}$.



Research Questions

Generalizing the LZ Effect to the cases where δ is a purely imaginary number gives rise to

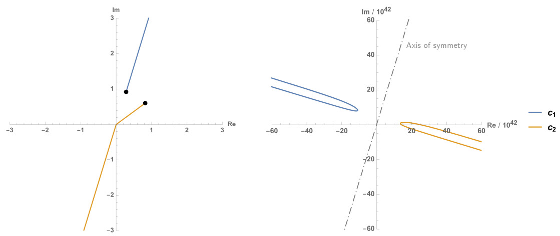
- Exceptional points at $\pm t_0$, where $t_0 = |\delta|/|\omega_0|$;
- Spontaneous PT-symmetry breaking.

What effect do they have on the dynamics of the system?

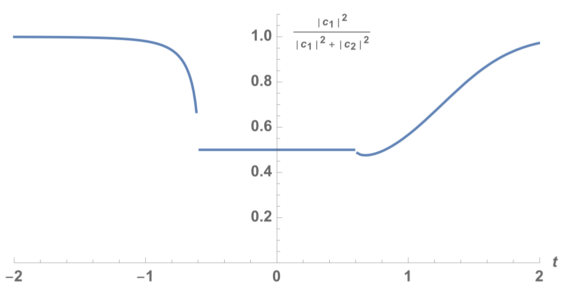
Results

Interesting numerical results are found for the coefficients of instantaneous eigenstates $c_1(t)$ and $c_2(t)$; we write

- $|\psi(t)\rangle = c_1(t)\,|\psi_+(t)\rangle + c_2(t)\,|\psi_-(t)\rangle$
- $|\psi_\pm(t)\rangle$ are eigenstates of $H(t)$.



Time evolution in the instantaneous basis, plot in the complex plane.
Left: $t < -t_0$. Right: $-t_0 < t < t_0$. Black dots denote initial states.



Time evolution of the normalized coefficient of the instantaneous eigenstate. The first-order derivative is discontinuous at EP's $\pm t_0$. Parameter: t_0=0.6.

Discussion

We showed analytically that the Hamiltonian of such form is always PT symmetric and that PT-symmetry breaking occurs only when $t<-t_0$ or $t>t_0$.

Mathematically, $|c_1(t)|=|c_2(t)|$ for $-t_0 < t < t_0$ is due to the fact that the phase difference between $c_1(t)$ and $c_2(t)$ is constantly π soon after the system is released at $t = -\infty$.

- This fact does not depend on the initial conditions.
- The conserved phase difference might be related to the PT symmetry of the system's Hamiltonian.

Future Directions:

- Seek a similar fact in many-body systems.
- View this fact in the broader context of dynamical phase transition [2].
- Relate to the Kibble-Zurek theory of topological defect production in phase transitions, whose connection to the usual LZ effect has been suggested in Ref.[3].

Acknowledgments

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